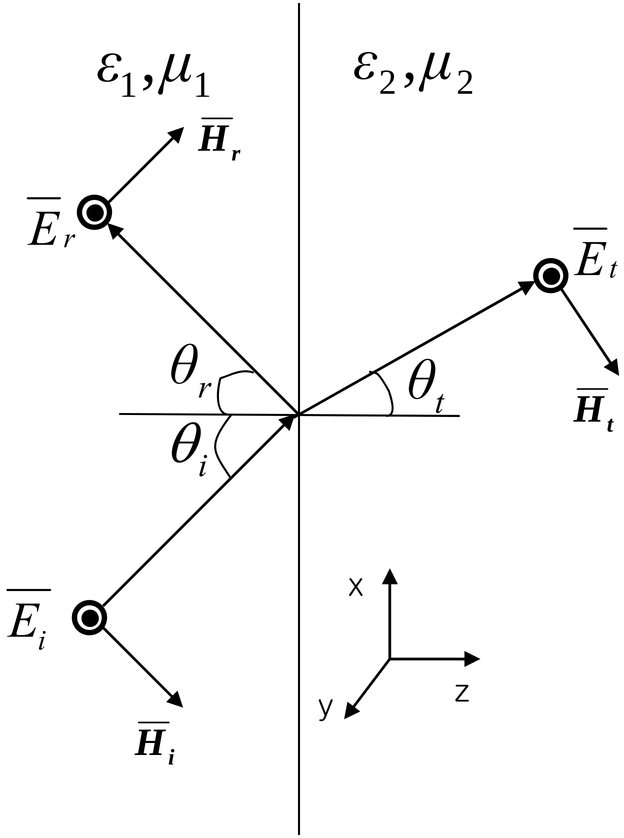
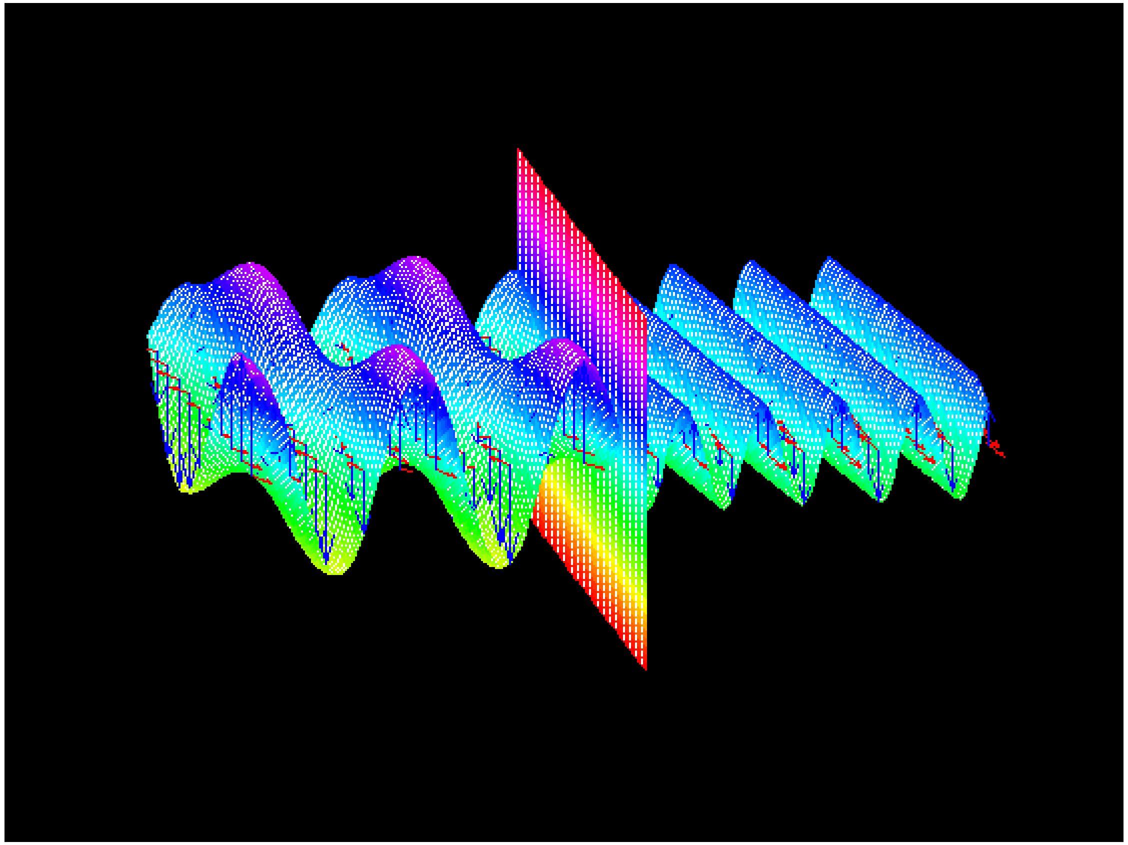
Lect. 4: Reflection and Transmission

(Kasap 1.5,1.6)

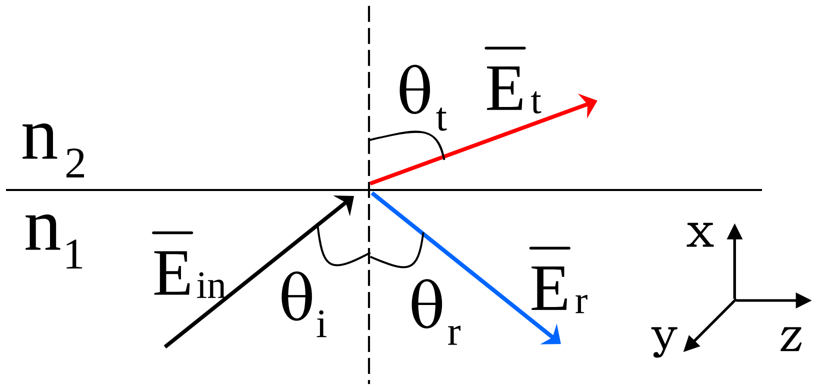




$$4\varepsilon_1 = \varepsilon_2,\ \ \mu_1 = \mu_2 \text{ and } \theta_i = 20^\circ$$

Lect. 4: Reflection and Transmission

(Assume E_{in} has only y-component: Perpendicular Polarization)



$$\overline{E}_{in} = \overline{y} E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y} E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y} E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_x = n_1 k_0 \cos\theta_i,\ k_z = n_1 k_0 \sin\theta_i$$

$$k_{rx} = n_1 k_0 \cos\theta_r, k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tx} = n_2 k_0 \cos\theta_t, k_{tz} = n_2 k_0 \sin\theta_t$$

Unknowns?

$$\theta_r, \theta_t, E_r, E_t$$

How to solve for Unknows?

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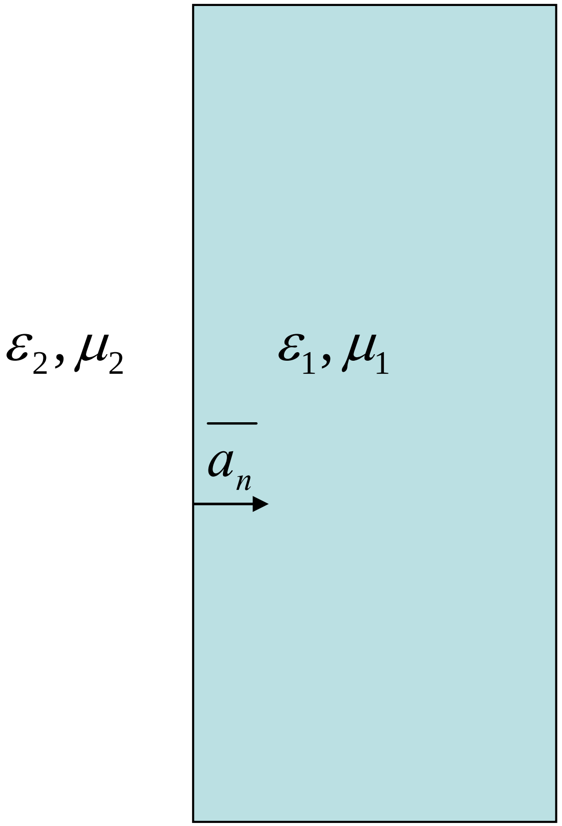
Lect. 4: Reflection and Transmission

Boundary Conditions: Constraints on E,H fields at a boundary.
Each Maxwell's Eq. provides one constraint on E or H.



$$\nabla \bullet \overline{D} = \rho,$$

$$D_{1,n} - D_{2,n} = \rho_s \Leftrightarrow \boxed{\varepsilon_1 E_{1,n} - \varepsilon_2 E_{2,n} = \rho_s}$$

$$\nabla \times \overline{E} = -\frac{d\overline{B}}{dt},$$

$$\boxed{E_{1,t} - E_{2,t} = 0}$$

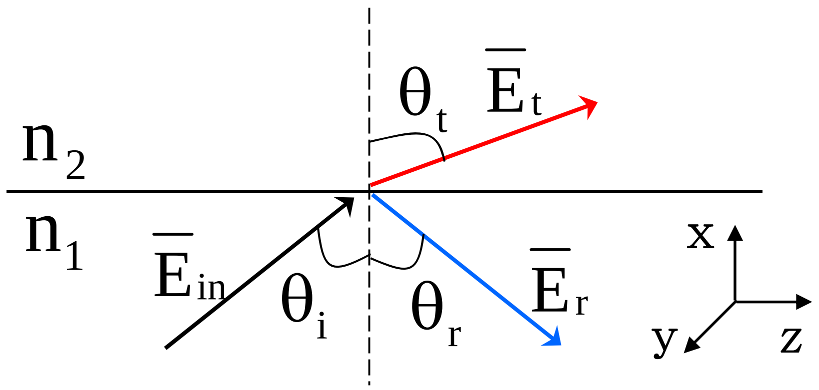
$$\nabla \bullet \overline{B} = 0,$$

$$B_{1,n} - B_{2,n} = 0 \Leftrightarrow \boxed{\mu_1 H_{1,n} - \mu_2 H_{2,n} = 0}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{d\overline{D}}{dt},$$

$$\boxed{\overline{a}_n \times (\overline{H}_1 - \overline{H}_2) = \overline{J}_s} \Leftrightarrow H_{1,t} - H_{2,t} = J_s$$

Lect. 4: Reflection and Transmission



$$\bar{E}_{in} = \bar{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\bar{E}_r = \bar{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\bar{E}_t = \bar{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

Applying BC on E at $x = 0$,

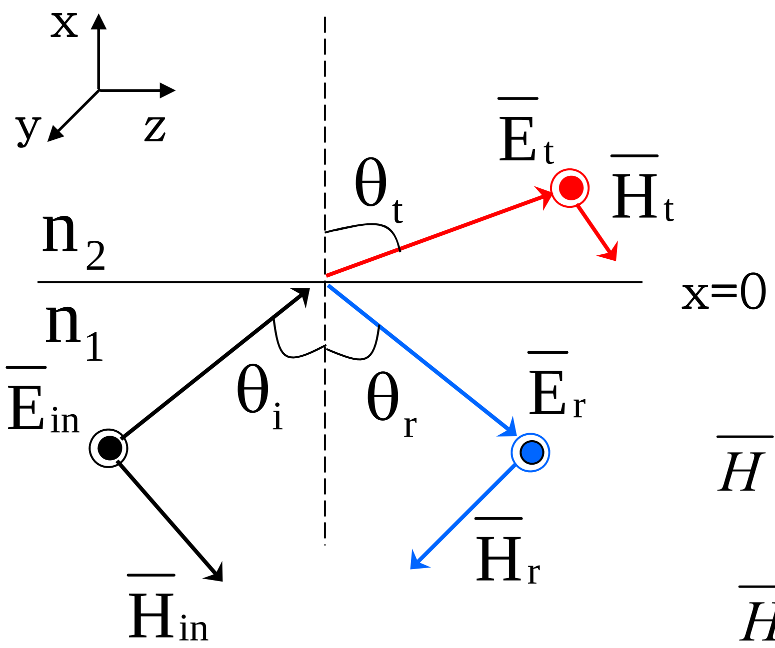
$$E_i e^{-jk_z \cdot z} + E_r e^{-jk_{rz} \cdot z} = E_t e^{-jk_{tz} \cdot z}$$

=> $k_z = k_{rz} = k_{tz}$ and $E_i + E_r = E_t$

$$\therefore n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$$

=> $\theta_i = \theta_r$ and $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's Law)

Lect. 4: Reflection and Transmission



$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$\overline{H}_{in} = \left(-\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right) E_i/\eta_1 \, e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \left(-\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right) E_r/\eta_1 \, e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \left(-\overline{x}\sin\theta_t + \overline{z}\cos\theta_t\right) E_t/\eta_2 \, e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on H at x=0

$$\left(\cos\theta_i\right) E_i/\eta_1 + \left(-\cos\theta_i\right) E_r/\eta_1 = \left(\cos\theta_t\right) E_t/\eta_2$$

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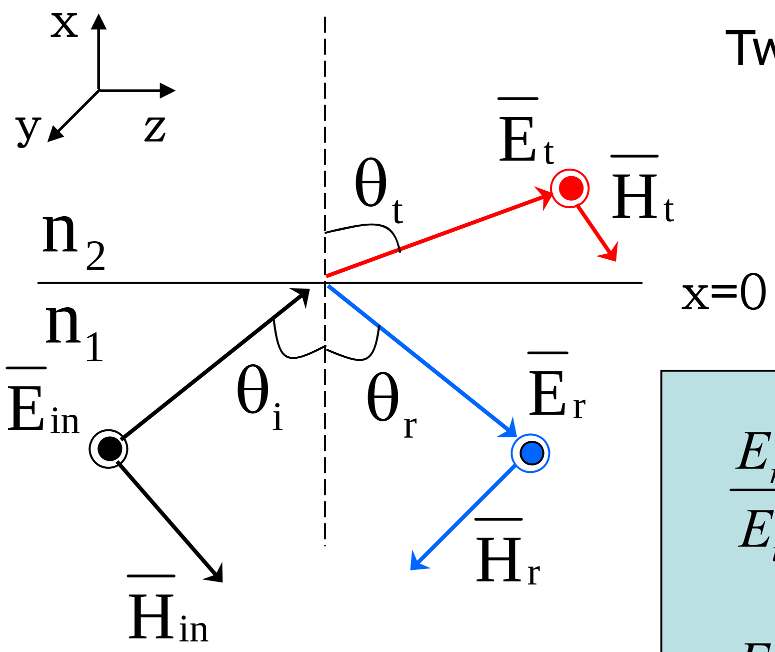
Lect. 4: Reflection and Transmission



Two simultaneous equations:

$$E_i + E_r = E_t$$

$$(\cos\theta_i)\, E_i/\eta_1 + (-\cos\theta_i)\, E_r/\eta_1 = (\cos\theta_t)\, E_t/\eta_2$$

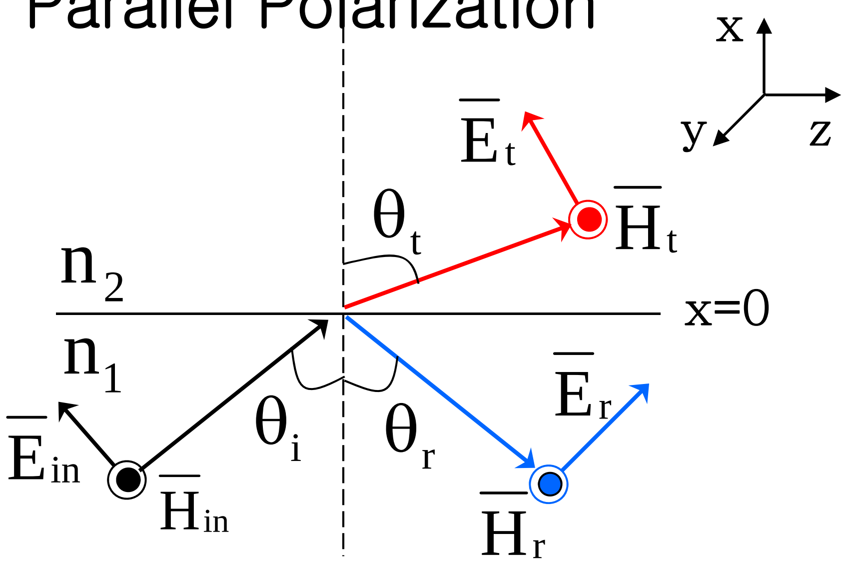
$$\frac{E_r}{E_i} = r_\perp = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$\frac{E_t}{E_i} = t_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad (n = \frac{n_2}{n_1})$$

$\theta_r, \theta_t, E_r, E_t$ are all solved!

Lect. 4: Reflection and Transmission

Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

$$\overline{E}_i = \left(\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\cdot \eta_1 H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

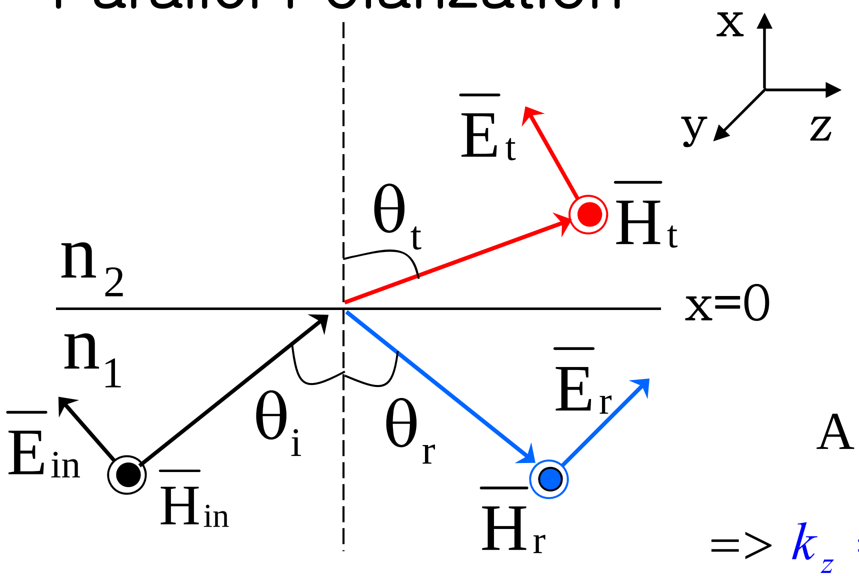
$$\theta_r, \theta_t, H_r, H_t = ???$$

$$\overline{E}_r = \left(\overline{x}\sin\theta_r + \overline{z}\cos\theta_r\right)\cdot \eta_1 H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \left(\overline{x}\sin\theta_t - \overline{z}\cos\theta_t\right)\cdot \eta_2 H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Lect. 4: Reflection and Transmission

Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on H at $x = 0$,

=> $k_z = k_{rz} = k_{tz}$ and $H_i + H_r = H_t$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

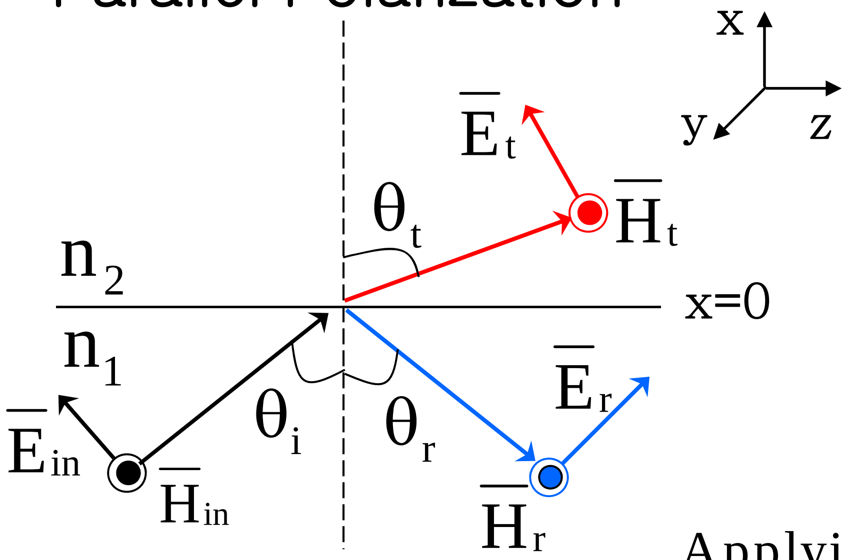
$$k_{tz} = n_2 k_0 \sin\theta_t$$

$$\therefore n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$$

=> $\theta_i = \theta_r$ and $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's Law)

Lect. 4: Reflection and Transmission

Parallel Polarization



$$\overline{E}_{in} = \left(\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\cdot\eta_1 H_i e^{-jk_x\cdot x} e^{-jk_z\cdot z}$$

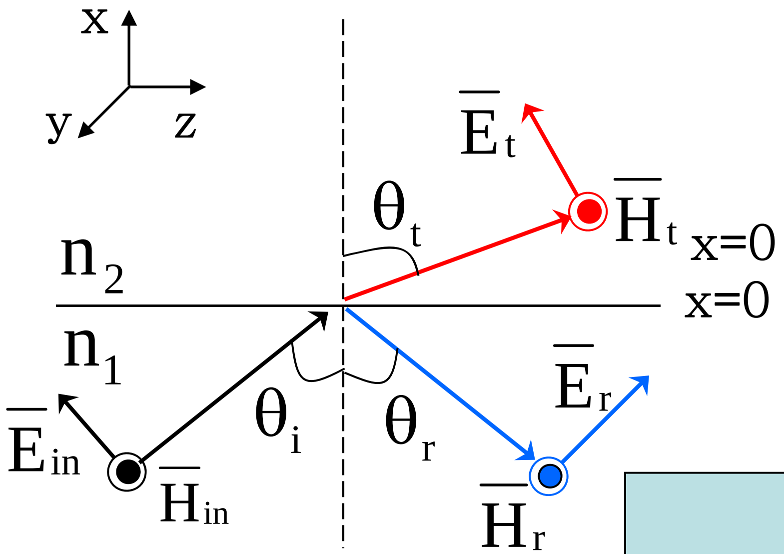
$$\overline{E}_r = \left(\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right)\cdot\eta_1 H_r e^{+jk_{rx}\cdot x} e^{-jk_{rz}\cdot z}$$

$$\overline{E}_t = \left(\overline{x}\sin\theta_t - \overline{z}\cos\theta_t\right)\cdot\eta_2 H_t e^{-jk_{tx}\cdot x} e^{-jk_{tz}\cdot z}$$

Applying BC on E at $x = 0$,

$$-\cos\theta_i\cdot\eta_1 H_i + \cos\theta_i\cdot\eta_1 H_r = -\cos\theta_t\cdot\eta_2 H_t$$

Lect. 4: Reflection and Transmission



Two simultaneous equations:

$$H_i + H_r = H_t$$

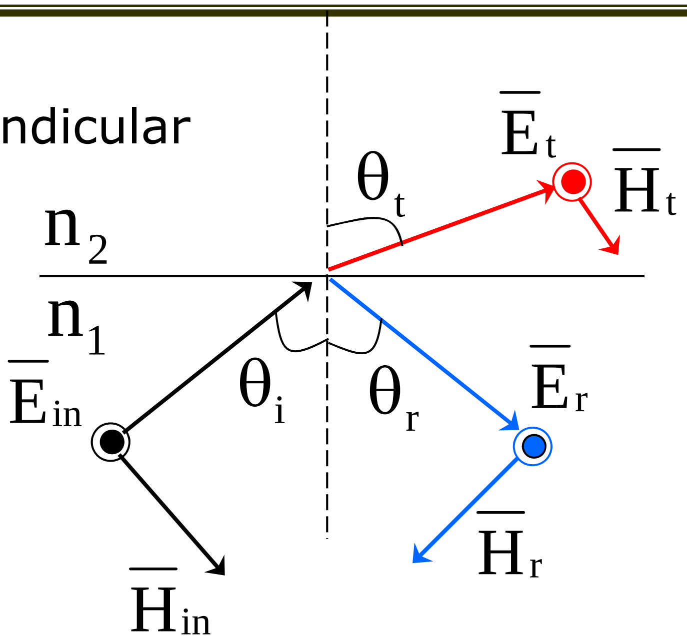
$$-\cos\theta_i \cdot \eta_1 H_i + \cos\theta_i \cdot \eta_1 H_r = -\cos\theta_t \cdot \eta_2 H_t$$

$$r_\parallel = -\frac{H_r}{H_i} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

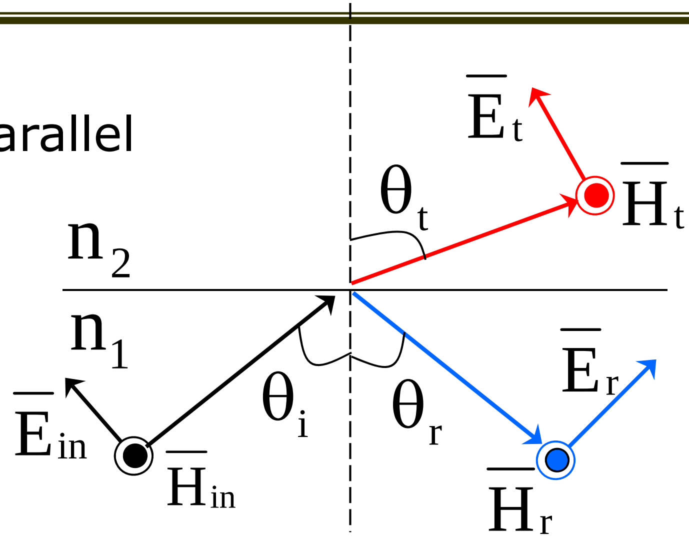
$$t_\parallel = \frac{\eta_1 H_t}{\eta_2 H_i} = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i} \qquad \left(n = \frac{n_2}{n_1}\right)$$

Lect. 4: Reflection and Transmission

Perpendicular



Parallel



$$r_\perp = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$r_\parallel = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_\parallel = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

Lect. 4: Reflection and Transmission

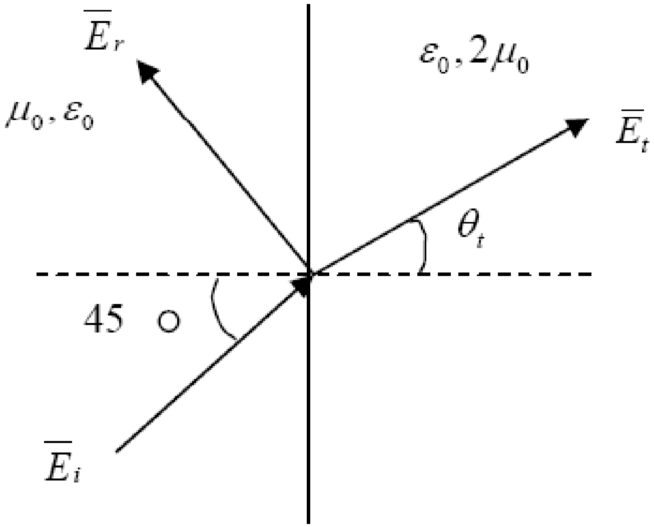
Homework: Due on 9/13

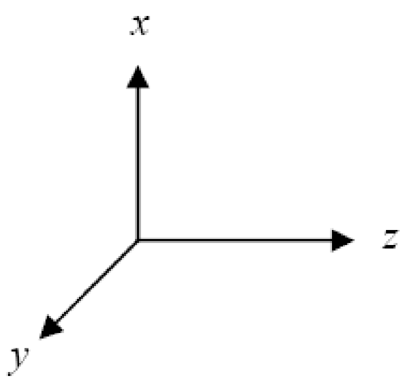
An EM wave is incident on dielectric interface as shown in the figure. There are no surface currents on the interface.

(a) Determine the angle of transmission.

(b) Derive an equation that relates r and t from the boundary condition on $\overline{H}$.

(c) Determines the numerical ratio of reflected power to incident power.





$$\overrightarrow{E}_i = \overline{y} e^{-jk_{1,x}x} e^{-jk_{1,z}Z}$$

$$\overrightarrow{E}_r = \overline{y} r e^{-jk_{1,x}x} e^{+jk_{1,z}z}$$

$$\overrightarrow{E}_t = \overline{y} t e^{-jk_{2,x}x} e^{-jk_{2,z}z}$$